13.0 EQUATIONS (senior two work)

		CONTENT	COMMENTS
		Objectives: To • describe an equation. • describe simultaneous equations. • solve simultaneous. • apply solution of simultaneous equations. • describe quadratic equations. • solve quadratic equations. • apply solution of quadratic equations.	
		Key words. equation, roots, solution or solving, linear simultaneous equation, elimination, substitution, quadratic equation, completing squares.	
	13.1	**Equation.** An equation is formed by equating an algebraic expression to another one or to zero. An equation in which the highest power is: (i) one is called **linear equation.** (ii) two is called **quadratic equation**. For example, $y = 2x + 1, x + y = 3 \; and \; 3x + 1 = 0$ are some linear equations. While $x^2 + 2x + 1 = 0$ and $x^2 + y^2 = 25$, are some quadratic equations. In all cases letters are used to represent **unknowns** whose values are to be determined. The process of **finding the values of the unknowns is solving**. The values determined are **solutions or roots** of a given equation.	
	13.2	**Simultaneous equations.** Two or more equations with two or more unknowns whose values satisfy all the equations given simultaneously. Here, the work will be restricted to two simultaneous equations with two unknowns. For example, the following are linear simultaneous equations with two unknowns. (a) $x + y = 3$ $x - y = 1$ having unknowns *x* and *y*. Solutions are *x* =2, *y* = 1 (b) $2a + b = 1$ $a + 2b = 8$ having unknowns *a* and *b*. Solutions are *a* = -2, *b* = 5 (c) $3u - v = 4$ $u - v = 2$ having unknowns *u* and *v*. Solutions are *u* =1, *v* = -1 Solving simultaneous equations. **<u>(a) Elimination method.</u>**	

In this method, any of the two unknowns given is eliminated by adding or subtracting the two equations using the rule of **SSS** and **DSA**.

Where **SSS** means **S**ame **S**ign **S**ubtract and **DSA** means **D**ifferent **S**ign **A**dd

Example 1. *Solve the simultaneous equations:*

$$x + y = 3$$
$$x - y = 1$$

Let

$$x + y = 3 \ldots\ldots\ldots\ldots(1)$$
$$x - y = 1 \ldots\ldots\ldots\ldots..(2)$$

To eliminate y, (the y terms have different signs). So, DSA can be applied. That is to say (1) + (2).

$$\frac{\begin{array}{r} x+y=3 \\ +(x-y=1) \end{array}}{2x+0=4}$$

$$2x = 4$$

$$\frac{2x}{2} = \frac{4}{2}$$

$$\therefore x = 2$$

Substituting $x = 2$ into equation (2),

$$2 - y = 1$$
$$\therefore y = 1$$

The solution is $\boldsymbol{x = 2}$ and **y = 1.**

Note:
When the coefficients of the unknowns in both equations are not the same, there is need to multiply either one equation or both equations by convenient values before applying the SSS and DSA rules.

Example 2. *Solve the simultaneous equations:*

$$2a + b = 1$$
$$a + 2b = 8$$

Let

$$2a + b = 1 \ldots\ldots\ldots\ldots(1)$$
$$a + 2b = 8 \ldots\ldots\ldots\ldots..(2)$$

To eliminate *a*, (the *a* terms have same signs but different coefficients). So, equation (2) is multiplied by 2 before the SSS rule is applied.
That is to say $2 \times (2) - (1)$

But first $2 \times (2) = 2 \times (a + 2b = 8) = 2a + 4b = 16$

$$\begin{array}{r} 2a+4b=16 \\ -(2a+b=1) \\ \hline 0+3b=15 \end{array}$$

$$3b = 15$$

$$\frac{3b}{3} = \frac{15}{3}$$

$$\therefore b = 5$$

Substituting b = 5 into equation (1),

$$2a + 5 = 1$$
$$2a = -4$$

$$\frac{2a}{2} = \frac{-4}{2}$$

$$\therefore a = -2$$

The solution is a = **-2** and b = **5.**

Example 3. *Solve the simultaneous equations:*

$$3u - v = 4$$
$$u - v = 2$$

Let

$$3u - v = 4 \ldots\ldots\ldots\ldots(1)$$
$$u - v = 2 \ldots\ldots\ldots\ldots(2)$$

To eliminate u, (the u terms have same signs but different coefficients). So, equation (2) is multiplied by 3 before the DSA rule is applied.

That is to say $3\times(2) - (1)$

But first $3\times(2) = 3 \times (u - v = 2) = 3u - 3v = 6$

$$\begin{array}{r} 3u-3v=6 \\ -(3u-v=4) \\ \hline 0-2v=2 \end{array}$$

$$-2v = 2$$

$$\frac{-2v}{-2} = \frac{2}{-2}$$

$$\therefore v = -1$$

Substituting v = -1 into equation (1),

$$3u - (-1) = 4$$

$$3u = 3$$

$$\frac{3u}{3} = \frac{3}{3}$$

$$\therefore u = 1$$

The solution is *u* = **1** and *v* = **-1.**

Note.

There is no rule as to which of the unknowns has to be eliminated first.

(b) Substitution Method

In this method, any of the equations is chosen and one of the unknowns expressed in terms of the other unknown.

It is then substituted out in the other equation.

Example 4.

Solve the simultaneous equations

$$5x + y = 7$$
$$3x - y = 1$$

Let

$$5x + y = 7 \ldots\ldots\ldots..(1)$$
$$3x - y = 1 \ldots\ldots\ldots..(2)$$

From equation (2), $y = 3x - 1$

Substituting for y in equation (1)

$$5x + (3x - 1) = 7$$

$$5x + 3x - 1 = 7$$

$$8x = 8$$
$$\frac{8x}{8} = \frac{8}{8}$$

$$\therefore x = 1$$

Substituting *x* = 1 into equation (1), we have

$$5 \times 1 + y = 7$$
$$5 + y = 7$$
$$y = 2$$

The solution is *x* = **1** and *y* = ***2.***

NOTE: When choosing the unknown to substitute, it is convenient to take the one with the lowest coefficient.

Example 5.

Solve the simultaneous equations

		$$2a + 3b = 1$$ $$a - b = 3$$ Let $$2a + 3b = 1 \ldots\ldots\ldots..(1)$$ $$a - b = 3 \ldots\ldots\ldots..(2)$$ From equation (2), $a = 3 + b$ Substituting for a in equation (1) $$2(3 + b) + 3b = 1$$ $$6 + 2b + 3b = 1$$ $$5b = -5$$ $$\frac{5b}{5} = \frac{-5}{5}$$ $$\therefore b = -1$$ Substituting $b = -1$ into equation (2), $$a - (-1) = 3$$ $$a + 1 = 3$$ $$a = 2$$ The solution is $b = -1$and $a = 2$	Two other methods are to be introduced later in the course. Matrix method Graphical method.
	13.3	**Applying solutions of simultaneous equations.** ***Example 6*** *Two pencils and three pens cost sh. 4,000. Three pencils and one pen cost sh. 2,500. Find the cost of one pencil and one pen.* Let the cost of a pencil be *c* and that of a pen be *p*. Then $$2c + 3p = 4{,}000 \ldots\ldots\ldots\ldots(1)$$ $$3c + p = 2{,}500 \ldots\ldots\ldots\ldots(2)$$ To eliminate *p*, (the *p* terms have same signs but different coefficients). So, equation (2) is multiplied by 3 before the DSA rule is applied.	

That is to say $3\times(2)-(1)$

But first $3\times(2) = 3\times(3c+p=2{,}500) = 9c+3p = 7{,}500$

$$\begin{array}{r} 9c+3p=7{,}500 \\ -(c+3p=4{,}000) \\ \hline 7c+0=3500 \end{array}$$

$$7c = 3{,}500$$

$$\frac{7c}{7} = \frac{3{,}500}{7}$$

$$\therefore c = 500$$

Substituting $c = 500$ into equation (1),

$$2\times 500 + 3p = 4{,}000$$
$$1{,}000 + 3p = 4{,}000$$
$$3p = 3{,}000$$

$$\frac{3p}{3} = \frac{3{,}000}{3}$$

$$\therefore p = 1{,}000$$

Each pencil costs sh 500. Each pen costs sh1,000

Example 7
Two vans and one bus can carry fifty students. One van and two buses can carry seventy students. If they are all filled to capacity, find the seat capacity of each.

Let the seat capacity of a van be *v* and that of a bus be *b*.

Then

$$2v + b = 50 \ldots\ldots\ldots\ldots (1)$$
$$v + 2b = 70 \ldots\ldots\ldots\ldots (2)$$

From equation (2), $v = 70 - 2b$

Substituting for *v* in equation (1)

$$2(70-2b) + b = 50$$

$$140 - 4b + b = 50$$

$$-3b = -90$$
$$\frac{-3b}{-3} = \frac{-90}{-3}$$

$$\therefore b = 30$$

Substituting $b = 30$ into equation (2),

$$v + 2(30) = 70$$
$$v + 60 = 70$$
$$v = 10$$

Therefore, a bus has 30 seats and a van has 10 seats.

Example 8
In two years, Gundi will be three times as old as Fundi. In five years, Gundi will be twice as old as Fundi. Find their current ages.
Let the current age of Gundi be g and the current age of Fundi be f.

	Gundi	Fundi	Relation between of the ages
Current age	g	f	
Age in two years	$g + 2$	$f + 2$	$g + 2 = 3(f + 2)$
Age in five years	$g + 5$	$f + 5$	$g + 5 = 2(f + 5)$

The relationships can be simplified by expansion and collection of like terms.

Thus,
$$g + 2 = 3(f + 2)$$
$$g + 2 = 3f + 6$$
$$g - 3f = 4 \ldots\ldots\ldots..(1)$$

And
$$g + 5 = 2(f + 5)$$
$$g + 5 = 2f + 10$$
$$g - 2f = 5 \ldots\ldots\ldots.(2)$$

Eliminating g by equation (2) – equation (1)
$$\frac{\begin{array}{c} g-2f=5 \\ -(g-3f=4) \end{array}}{0+f=1}$$
$$\therefore f = 1$$

Substituting $f = 1$ into equation (1)
$$g - 3 \times 1 = 4$$
$$g - 3 = 4$$
$$g = 7$$

Therefore Gundi is 7 years old and Hundi is 1year old

Example 9.
Find the point of intersection of the lines $y = 2x + 3$ *and* $y = 4x + 1$.

At the point of intersection, the values of *x* and *y* must satisfy both equations at the same time.
They have to be solved as simultaneous equations.

Let

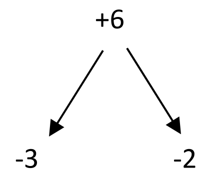
		$y = 2x + 3 \ldots \ldots \ldots \ldots (1)$ $y = 4x + 1 \ldots \ldots \ldots \ldots (2)$ From equation (1) $y = 2x + 3$ Substituting for y in equation (2) $2x + 3 = 4x + 1$ $-2x = -2$ $x = 1$ Substituting $x = 1$ into equation (1) $y = 2 \times 1 + 3$ $y = 5$ Therefore the point of intersection is (1, 5).	
	13.4	**Quadratic equations.** You may recall **quadratic expressions** in the topic of expansion and factorisation. If a quadratic expression is equated to zero, a **quadratic equation is obtained.** For example, $x^2 + x - 2 = 0$, $x^2 + 3x = 0$ and $x^2 + y^2 - 16 = 0$ are quadratic equations The work here will involve quadratic equations with one unknown. **Solving** a quadratic equation means finding the possible values of the unknown letter (or **roots**) that satisfy such equation. **(a) By factorisation.** **A property of zero** is used. **Given two numbers P and Q, and P×Q = 0,** **Then, either P =0 or Q = 0 or both are zero.** **……….F 13.1** **Quadratic equations with two terms**. Notice that these are **easy to factorise**. ***Example 10*** *Solve the quadratic equation* $x^2 + 5x = 0$ $x^2 + 5x = 0$ Factorising left hand side, $x(x + 5) = 0$ Using the property of zero, Either $x = 0$ Or $(x + 5) = 0$ so, $x = -5$	

Quadratic equations with three terms.

The skills of factorizing three term quadratic expressions apply.

Example 11
Solve the quadratic equation $x^2 - 5x + 6 = 0$.

The coefficient of the first term is 1. The constant term is 6.
Their product is 1 × 6 = 6
The coefficient of the second term is -5.
Which two numbers would add to -5 and have product of 6?



$$x^2 - 5x + 6 = 0$$
$$x^2 - 2x - 3x + 6 = 0$$
$$(x^2 - 2x) + (-3x + 6) = 0$$
$$x(x - 2) - 3(x - 2) = 0$$
$$(x - 2)(x - 3) = 0$$

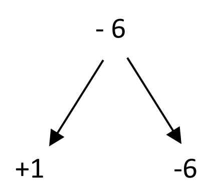
Using the property of zero,

Either $(x - 2) = 0 \Rightarrow x = 2$

Or $(x - 3) = 0 \Rightarrow x = 3$

Example 12
Solve the quadratic equation $2x^2 - 5x - 3 = 0$.

The coefficient of the first term is 2. The constant term is -3.
Their product is 2 × -3 = - 6
The coefficient of the second term is -5
Which two numbers would add to -5 and have product of - 6?



$$2x^2 - 5x - 3 = 0$$
$$2x^2 + x - 6x - 3 = 0$$
$$(2x^2 + x) + (-6x - 3) = 0$$
$$x(2x + 1) - 3(2x + 1) = 0$$
$$(2x + 1)(x - 3) = 0$$

Using the property of zero,

Either $(2x+1) = 0 \Rightarrow x = -\frac{1}{2}$

Or $(x-3) = 0 \Rightarrow x = 3$

Some solutions may require rearranging to obtain a quadratic equation.

Example 13.

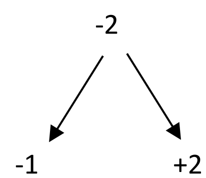
Solve $\frac{x+1}{x} = \frac{x+3}{2}$

By cross multiplication,

$$2(x+1) = x(x+3)$$
$$2x+2 = x^2+3x$$

Rearranging, $x^2 + x - 2 = 0$

The coefficient of the first term is 1. The constant term is -2.
Their product is 1 × -2 = -2
The coefficient of the second term is +1.
Which two numbers would add to +1 and have product of -2 ?



$$x^2 + x - 2 = 0$$
$$x^2 - x + 2x - 2 = 0$$
$$(x^2 - x) + (2x - 2) = 0$$
$$x(x-1) + 2(x-1) = 0$$
$$(x-1)(x+2) = 0$$

Using the property of zero,

Either $(x-1) = 0 \Rightarrow x = 1$

Or $(x+3) = 0 \Rightarrow x = -3$

Formation of quadratic equations whose roots are given.
Since one way of solving quadratic equations is by factorization, expansion can be applied to form an equation from the roots.

Example 14.

Form a quadratic equation whose roots are -2 and +3.

If the roots are -2 and +3, it means

Either $x = -2$	Or $x = +3$
$x + 2 = -2 + 2$	$x - 3 = +3 - 3$
$\Rightarrow (x+2) = 0$	$\Rightarrow (x-3) = 0$

Multiplying, $(x+2) \times (x-3) = 0 \times 0 = 0$

Therefore, $x(x-3) + 2(x-3) = 0$

$$x^2 - 3x + 2x - 6 = 0$$
$$x^2 - x - 6 = 0 \text{ is the equation.}$$

Learners should solve the final equation by factorization.

<u>(b) By completing of squares.</u>

$$(x + 2)^2 = (x + 2)(x + 2)$$
$$= x^2 + 4x + 4 \text{ is a perfect square.}$$

But $x^2 + 4x$ is not a perfect square. To make it a perfect square ,4 has to be added to it. This is called completing the square.

To solve the quadratic equation $ax^2 + bx + c = 0$ by completing the square the following steps may be useful.

Step 1 The equation is divided by the coefficient of the first term.

$$\frac{a}{a}x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

Note. If the coefficient of the first term is 1, there will be no need to divide. So, this step can be skipped.

Step 2 The new equation is then written in a form such that only the constant term is on the right-hand side.

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

Step 3 $(\mathit{Half\ the\ coefficient\ of\ the\ x\ term\ of\ the\ new\ equation})^2$ or $\left(\frac{b}{2a}\right)^2$, is added to the left-hand side to complete the square there and added to the right-hand side to balance the equation.

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$

Step 4 The left-hand side of the equation is factorized into a perfect square.

$$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$

Step 5 The square roots on both sides of the equation are taken.

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{-\frac{c}{a} + \left(\frac{b}{2a}\right)^2}$$
$$x + \frac{b}{2a} = \pm\sqrt{-\frac{c}{a} + \left(\frac{b}{2a}\right)^2}$$

Step 6 The values of x are determined.

$$x = -\frac{b}{2a} + \sqrt{-\frac{c}{a} + \left(\frac{b}{2a}\right)^2} \text{ or } x = -\frac{b}{2a} - \sqrt{-\frac{c}{a} + \left(\frac{b}{2a}\right)^2}$$

Example 15.
Solve the quadratic equation $x^2 + 6x + 8 = 0$ by completing squares.

Since the coefficient of the first term is 1, there is no need to divide.
Moving the constant (or number) term to the right-hand side,

$$x^2 + 6x + 8 - 8 = 0 - 8$$

		$$x^2+6x=-8$$ The coefficient of the second term is 6. Dividing this by 2 gives $\frac{6}{2}=3$. 3^2 is added to both sides of the equation. $$x^2+6x+3^2=-8+3^2$$ Factorizing the right-hand side into a perfect square. $$(x+3)^2=-8+9$$ $$(x+3)^2=1$$ Taking the square root of each side, $$\sqrt{(x+3)^2}=\sqrt{1}$$ $$x+3=\pm 1 \quad \text{(since } (-1)^2=1 \text{ and } 1^2=1\text{)}$$ Finding the values of x. For $x+3=+1$: $x+3-3=1-3$, $x=-2$ For $x+3=-1$: $x+3-3=-1-3$, $x=-4$ ***Example 16.*** *Solve the quadratic equation* $3x^2-5x+2=0$ *by completing squares.* Since the coefficient of the first term is 3, each of the terms is divided by 3. $$\frac{3}{3}x^2-\frac{5}{3}x+\frac{2}{3}=0$$ $$x^2-\frac{5}{3}x+\frac{2}{3}=0$$ Moving the constant (or number) term to the right-hand side, $$x^2-\frac{5}{3}x+\frac{2}{3}-\frac{2}{3}=0-\frac{2}{3}$$ $$x^2-\frac{5}{3}x=-\frac{2}{3}$$ The coefficient of the second term is $-\frac{5}{3}$. Dividing this by 2 gives $-\frac{5}{3}\div 2=-\frac{5}{6}$. $\left(-\frac{5}{6}\right)^2$ is added to both sides of the equation. $$x^2-\frac{5}{3}x+\left(-\frac{5}{6}\right)^2=-\frac{2}{3}+\left(-\frac{5}{6}\right)^2$$ Factorizing the right-hand side into a perfect square. $$\left(x-\frac{5}{6}\right)^2=-\frac{2}{3}+\frac{25}{36}$$ $$\left(x-\frac{5}{6}\right)^2=-\frac{24}{36}+\frac{25}{36}$$ $$\left(x-\frac{5}{6}\right)^2=\frac{1}{36}$$ Taking the square root of each side,	Learners should try to solve examples 15,16 and 17 by method of factorization. This should help in making an important observation at the end.

$$\sqrt{\left(x-\frac{5}{6}\right)^2}=\sqrt{\frac{1}{36}}$$

$$x-\frac{5}{6}=\pm\frac{1}{6}$$

Finding the values of *x* .

For $x-\frac{5}{6}=+\frac{1}{6}$	For $x-\frac{5}{6}=-\frac{1}{6}$
$x=\frac{5}{6}+\frac{1}{6}$	$x=\frac{5}{6}-\frac{1}{6}$
$x=1$	$x=\frac{4}{6}=\frac{2}{3}$

Example 17.
Solve the quadratic equation $x^2+3x+1=0$ *by completing squares.*

Since the coefficient of the first term is 1, there is no need to divide.

Moving the constant (or number) term to the right-hand side,

$$x^2-3x+1-1=0-1$$

$$x^2-3x=-1$$

The coefficient of the second term is -3.
Dividing this by 2 gives $-3\div 2=-\frac{3}{2}$.
$\left(-\frac{3}{2}\right)^2$ is added to both sides of the equation.

$$x^2-3x+\left(-\frac{3}{2}\right)^2=-1+\left(-\frac{3}{2}\right)^2$$

Factorizing the right-hand side into a perfect square.

$$\left(x-\frac{3}{2}\right)^2=-1+\frac{9}{4}$$

$$\left(x-\frac{3}{2}\right)^2=-\frac{4}{4}+\frac{9}{4}$$

$$\left(x-\frac{3}{2}\right)^2=\frac{5}{4}$$

Taking the square root of each side,

$$\sqrt{\left(x-\frac{3}{2}\right)^2}=\sqrt{\frac{5}{4}}$$

$$x-\frac{3}{2}=\pm 1.1180(\textit{by using a calculator})$$

Finding the values of *x* .

For $x-\frac{3}{2}=+1.1180$	For $x-\frac{3}{2}=-1.1180$
$x=\frac{3}{2}+1.1180$	$x=\frac{3}{2}-1.1180$
$x=2.6180$	$x=0.382$

Notice that the method of factorization does not work for one of the examples 15, 16 and 17.

So, the method of completing squares is more powerful than the method of factorization.

(b) By using the quadratic formula.

Solving by completing squares provides the basis for deriving the quadratic formula.

Recall the steps outlined in the solution of $ax^2+bx+c = 0$ by completing of squares.

Picking off the flow of the steps from step 5 where the square roots on both sides of the equation are taken.

$$\sqrt{\left(x+\frac{b}{2a}\right)^2} = \pm\sqrt{-\frac{c}{a}+\left(\frac{b}{2a}\right)^2}$$

$$x+\frac{b}{2a} = \pm\sqrt{-\frac{c}{a}+\left(\frac{b}{2a}\right)^2}$$

$$x+\frac{b}{2a} = \pm\sqrt{-\frac{c}{a}+\frac{b^2}{4a^2}}$$

$$x+\frac{b}{2a} = \pm\sqrt{-\frac{4ac}{4a^2}+\frac{b^2}{4a^2}}$$

$$x+\frac{b}{2a} = \pm\sqrt{\frac{b^2-4ac}{4a^2}}$$

$$x+\frac{b}{2a} = \pm\frac{\sqrt{b^2-4ac}}{\sqrt{4a^2}}$$

$$x+\frac{b}{2a}-\frac{b}{2a} = -\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a} \text{..............} \textbf{\textit{F 13.2}}$$

Example 18 Using the formula, solve x^2 - 12x + 27 = 0

By comparing with the standard form $ax^2+bx+c = 0$.

$a = 1, \quad b = -12 \quad and \quad c = 27$

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-(-12)\pm\sqrt{(-12)^2-4\times1\times27}}{2\times1}$$

$$x = \frac{12\pm\sqrt{144-108}}{2}$$

$$x = \frac{12\pm\sqrt{36}}{2}$$

Either $x = \frac{12+6}{2}$ or $x = \frac{12-6}{2}$

$\therefore\ x = 9$ or 3

Learners should take care of the handling of numbers with negative signs. e.g. -12(here) Quite often, they have been sources of errors.

Example 19 Using the formula, solve $3x^2 + 1 = 4x$.

Rearranging into standard form, $\boldsymbol{3x^2 - 5x + 1 = 0}$.
By comparing with the standard form $ax^2+bx+c = 0$.
$a = 3$, $b = -5$ and $c = 1$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$
$$x = \frac{-(-5) \pm \sqrt{(-5)^2-4\times3\times1}}{2\times1}$$
$$x = \frac{5 \pm \sqrt{25-12}}{2}$$
$$x = \frac{5 \pm \sqrt{13}}{2}$$

Either $x = \frac{5+\sqrt{13}}{2}$ or $x = \frac{5-\sqrt{13}}{2}$
$\therefore\ x = 3.6180$ or 1.3820

(c) Graphical method.

(To be covered later in the course)

(d) Calculator method.

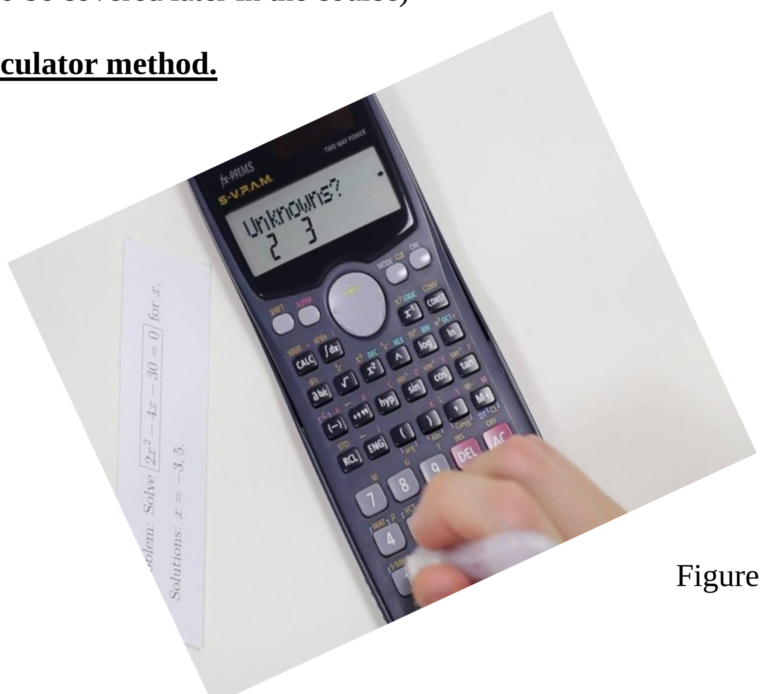


Figure 1

Different models of scientific calculators require different order of inputs.

It is important to find out the order of input on your personal calculator that will lead to the solution of quadratic equations.
Hereafter, this method can be used to verify our solutions from the previous methods.

13.5 **Applying solutions of quadratic equations.**

Many real-life problems requiring mathematical solutions end up with the formation and solution of quadratic equations.
In the formation of equations, letters are chosen and named at random. The quadratic equations would be formed in the letters chosen.

Some may require a sketch diagram or table to organize and visualize the problem.

Example 20.
50m of fencing material is to be used to fence off a rectangular area of $100m^2$. Find the possible dimensions of the area.

Let the width be x.
The length will then be $(50 - 2x) \div 2 = 25 - x$ as shown in figure 2.

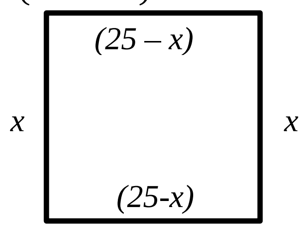


Figure 2

$$Area = length \times width$$

Substituting, $100 = (25 - x) \times x$

Opening the bracket, $100 = 25x - x^2$

Rearranging into standard form, $x^2 - 25x + 100 = 0$

By comparing with the standard form $ax^2+bx+c = 0$.

$a = 1$, $b = -25$ and $c = 100$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-25) \pm \sqrt{(-25)^2 - 4 \times 1 \times 100}}{2 \times 1}$$

$$x = \frac{25 \pm \sqrt{625 - 400}}{2}$$

$$x = \frac{25 \pm \sqrt{125}}{2}$$

Either $x = \frac{25+\sqrt{125}}{2}$ or $x = \frac{5-\sqrt{125}}{2}$

x = 18.0902 or -3.0902

Since there are no negative widths, x = -3.0902 is discarded.

Therefore, the width = 18.0902m and length = 25 – 18.0902= 6.9098m

Example 21.
A fencing material is to be used to fence off an area of $96m^2$. If the width is to be 4m shorter than the length, find the length of fencing material required.

Let the width be x.
The length will then be $(x + 4)$ as shown in figure 3.

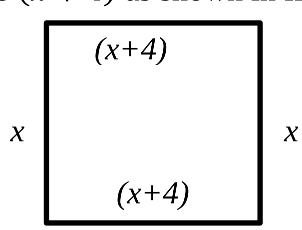


Figure 3

$$Area = length \times width$$

Substituting, $96 = (x + 4) \times x$

Opening the bracket, $96 = x^2 + 4x$

Rearranging into standard form, $x^2 + 4x - 96 = 0$

By comparing with the standard form $ax^2+bx+c = 0$.

$a = 1, \quad b = 4 \quad and \quad c = -96$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{4^2-4\times1\times(-96)}}{2\times1}$$

$$x = \frac{-4 \pm \sqrt{16+384}}{2}$$

$$x = \frac{-4 \pm \sqrt{400}}{2}$$

Either $x = \frac{-4+20}{2}$ or $x = \frac{-4-20}{2}$

$x = 8$ or -12

Since there are no negative widths, $x = -12$ is discarded.

Therefore, the width = 8m and length = 4 + 8= 12m

Total length required = 2× (length + width) = 2 × (8 + 12) = 40m

Example 22.

A lady initially bought petrol for sh 12,000. After a price increase of sh 1,000per litre, she can now buy 2 litres less for the same amount. Find the current price of petrol per litre.

Let the initial cost per litre be x and the initial number of litres be N.

	Cost per litre (sh.)	Number of litres	Amount spent (sh.)	Relating N and x.
Initially	x	N	12,000	$N = \frac{12,000}{x}$
Currently	$x + 1,000$	N-2	12,000	$N - 2 = \frac{12,000}{x+1,000}$

$$N = \frac{12,000}{x} \ldots\ldots\ldots\ldots(1)$$

$$N - 2 = \frac{12,000}{x+1,000} \ldots\ldots\ldots(2)$$

Substituting N into equation (2)

$$\left(\frac{12,000}{x}\right) - 2 = \frac{12,000}{x+1,000}$$

$$\left(\frac{12,000-2x}{x}\right) = \frac{12,000}{x+1,000}$$

Cross multiplying, $(12,000 - 2x)(x + 1,000) = 12,000 \times x$

Expanding, $12,000(x + 1,000) - 2x(x + 1,000) = 12,000 \times x$

$$12,000x + 12,000,000 - 2x^2 - 2,000x = 12,000x$$

$$12,000,000 - 2x^2 - 2,000x = 0$$

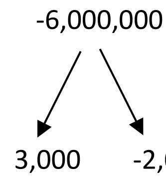
Multiplying through by -1

$$2x^2 + 2{,}000x - 12{,}000{,}000 = 0$$

Dividing through by 2,

$$x^2 + 1{,}000x - 6{,}000{,}000 = 0$$

Product = 6,000,000 and sum = 1,000



$$x^2 - 2{,}000x + 3{,}000x - 6{,}000{,}000 = 0$$
$$(x^2 - 2{,}000x) + (3{,}000x - 6{,}000{,}000) = 0$$
$$x(x - 2{,}000) + 3{,}000(x - 2{,}000) = 0$$
$$(x - 2{,}000)(x + 3{,}000) = 0$$
$$Either\ (x - 2{,}000) = 0\ or\ (x + 3{,}000) = 0$$
$$Either\ x = 2{,}000\ or\ x = -3{,}000$$

Discarding x = -3,000 leaves x = 2,000

Therefore, current price per litre = 2,000+1,000 = 3,000sh.

Example 23*(intersection of a quadratic curve and a line).*
Find the point of intersection of the curve $y = x^2 + 1$ *and the line* $y = x + 3$.

At the point of intersection, the values of *x* and *y* must satisfy both equations at the same time.

They have to be solved as simultaneous equations.

Let

$$y = x + 3 \dots\dots\dots\dots (1)$$
$$y = x^2 + 1 \dots\dots\dots\dots (2)$$

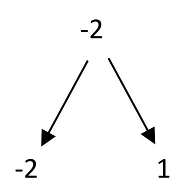
From equation (1) $\quad y = x + 3$

Substituting for y into equation (2)

$$x + 3 = x^2 + 1$$

Rearranging into standard form, $x^2 - x - 2 = 0$

Sum = -1 product = -2



$$x^2 + x - 2x - 2 = 0$$
$$(x^2 + x) + (-2x - 2) = 0$$
$$x(x + 1) - 2(x + 1) = 0$$
$$(x + 1)(x - 2) = 0$$
$$Either\ (x + 1) = 0\ or\ (x - 2) = 0$$
$$x = -1\ and\ x = 2$$

When $x = -1,\ y = -1 + 3 = 2 \ \Rightarrow (-1{,}2)$

When $x = 2,\ y = 2 + 3 = 5 \ \Rightarrow (2\ {,}\ 5)$

	13.6	*Exercise.*	

Exercise.

1. *Solve the following pairs of simultaneous equations by elimination.*

(a) $x + y = 4$, $x - y = 2$ (b) $3a + 2b = 5$, $5a - 2b = 3$ (c) $3u + v = -9$, $u - 2v = 4$

2. *Solve the following pairs of simultaneous equations by substitution*

(a) $x + 2y = 6$, $x - 3y = 1$ (b) $2a - b = 3$, $a + 2b = -1$ (c) $2u - v = 5$, $u - 2v = 2$

3. *Two packets of sugar and three packets of salt weigh 1300g. One packet of sugar and four packets of salt weigh 900g. Find the weight of each of the packets.*

4. *Three years ago, Trisha was six times as old as Maria. In five years' time, Trisha will be twice as old as Maria. Find their current ages.*

5. *If* $x : y = 2 : 1$ *and* $x + y = 6$, *find the values of x and y.*

6. *Find the point of intersection of the lines* $y = 2x + 1$ *and* $y = 3x - 2$.

7. *Solve the quadratic equation* $x^2 + x - 30 = 0$ *by factorization.*

8. *Solve the quadratic equation* $2x^2 + 3x + 1 = 0$ *by completing squares.*

9. *Find the values of x such that* $\frac{x^2+2x+1}{x+3} = 1$.

10. *Solve the quadratic equation* $2x^2 - x = 2$ *using the quadratic formula.*

11. *Solve the equation* $\frac{x+1}{x-1} = \frac{2x+2}{x}$.

12. *The difference between two adjacent sides of a rectangle is 2m.If the are of the rectangle is* $143m^2$, *find the length of the perimeter.*

13. *A man is currently three times as old as the daughter. In two years' time, the product of their ages will be 384. Find their current ages.*

14. *Given that* $121_n = 16$, *find n.*

15. *When four times a number is increased by 12, the result is equal to the square of the number. Find the number.*